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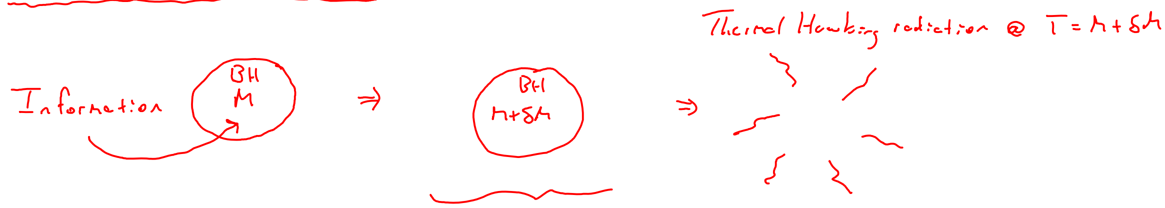
## Brief review of the BHIP




Maybe okay since information hidden behind horizon

Uh-oh, no more horizon!

If information lost: Non-unitary evolution which means quantum anything gets broken.
If Hawking radiation is wrong: Something off about effective field theory, equivalence principle, etc.

Partial resolution via $\underbrace{\text{AdS}}_{\text{almost anything gravity}} / \underbrace{\text{CFT}}_{\text{non-gravity QFT}}$ $\Rightarrow$ Unitarity must be preserved!

But how? While this, and his own similar argument, led Hawking to concede the bet... the details were still unclear.

So folks kept thinking.

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Since these ideas hover around entropy and information, let's sharpen these notions a bit.

Heuristically: Entropy is a measure of what we can know, but choose not to know about a system.
Entropy is a physically meaningful (and impactful) quantity because the process of gaining knowledge is achieved through physical investigation, i.e. detection and we know that this has physical consequences.

Classical Thermodynamic Entropy: For macroscopic systems (under reversible changes from eq to eq)

$$\underbrace{dE}_{\Delta\text{ energy}} = \overset{\text{temp}}{T}\underbrace{dS}_{\Delta\text{ entropy}} + \overset{\text{ang. pot.}}{\Omega}\underbrace{dJ}_{\Delta\text{ in ang mom}} + \overset{\text{elec. pot.}}{\Phi}\underbrace{dQ}_{\Delta\text{ in elec charge}}$$

$\underbrace{T\,dS}_{\text{heat transferred}}$

$E, T, S, \Omega, J, \Phi, Q$ are macro. state variables

Statistical Entropy: Given a coarse-grained description (in terms of average quantities only for example) there are usually many fine-grained resolutions compatible.
If the $i$th compatible resolution (microstate) has probability $p_i$ to be the true state of the system then $S_S \propto -\sum_i p_i \ln p_i$. Note for $p_1 = 1$, $p_{i\neq 1} = 0$
$S_S = 0$
The language we use is that for a coarse-grained description, there is an ensemble of resolutions such that $|\text{what we know}\rangle = \sum_i p_i\, |i\text{th resolution}\rangle$

Microcanonical: Fix $E$ & $N$ $\Rightarrow$ equal prob. $\Rightarrow$ $p_i = \frac{1}{\#}$ $\Rightarrow$ $S_S = k\sum_i \frac{1}{\#}\ln\# = k\ln\#$
For $\# = 1$ $\Rightarrow$ $S_S = 0$

Canonical: Fix $T$ & $N$ $\Rightarrow$ $p_i = \frac{1}{Z}e^{-E_i/kT}$ w/ $Z = \sum_i e^{-E_i/kT}$ so $\sum_i p_i = 1$

$S_S = k\ln Z + \frac{\langle E\rangle}{T}$ for $T\to 0$ (only $E_0$ contributes) $\Rightarrow$ $S_S \to 0$

In all cases $S$ is extensive, i.e. $S(A\cup B) = S(A) + S(B)$ (volume scaling)

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Von Neumann Entropy: In a quantum system there is a limit to how much we can possibly know, i.e. uncertainty, e.g. $[\hat{X}, \hat{p}_x] \neq 0$. This means in particular that even if we measure (know) everything allowed we can still be in a superposition $|\psi\rangle = \sum c_i |E_i\rangle$, but this is <u>the</u> state of the system so should have $S = 0$. $\Sigma$ note $|c_i|^2 = p_i$

To accomodate this define $\rho = |\psi\rangle\langle\psi|$ then $S_{VN} = -Tr\,\rho \ln \rho$

E.g. $|\psi\rangle = a|\uparrow\rangle_A |\downarrow\rangle_B + b|\downarrow\rangle_A |\uparrow\rangle_B$ w/ $|a|^2 + |b|^2 = 1$

$\rho = |\psi\rangle\langle\psi| = (a|\uparrow\rangle_A|\downarrow\rangle_B + b|\downarrow\rangle_A|\uparrow\rangle_B)(a^* {}_A\langle\uparrow| {}_B\langle\downarrow| + b^* {}_A\langle\downarrow| {}_B\langle\uparrow|)$

$= \begin{pmatrix} aa^* & ab^* \\ ba^* & bb^* \end{pmatrix} \Rightarrow \rho^2 = \rho$ (always for pure state)

$S_{VN} = -Tr\,\rho \ln \rho = -Tr\,\rho \ln \rho^2 = -Tr\,2\rho \ln \rho = -4\,Tr\,\rho \ln \rho \Rightarrow 1 = 4$ unless $S_{VN} = 0$

If on the other hand we do not measure everything we can, then the resolution of our course graining can be represented by a collection of (possibly superposition) states $|\psi\rangle_j$ and we then have what is equivalent to an ensemble but in quantum parlance called a "mixed state". We cannot describe it w/ a state vector, but the density matrix comes in handy here: $\rho = \sum_j p_j |\psi\rangle_j \langle\psi|_j$

Where things can get interesting is:

Now suppose we choose to know less by only knowing about subsystem A. To get to this description from $|\psi\rangle$ above, we trace over the state space of B. This leads to the reduced density matrix:

$\rho_A = Tr_B\,\rho = {}_B\langle\uparrow|\rho|\uparrow\rangle_B + {}_B\langle\downarrow|\rho|\downarrow\rangle_B = |b|^2 |\downarrow\rangle_A {}_A\langle\downarrow| + |a|^2 |\uparrow\rangle_A {}_A\langle\uparrow|$

$= \begin{pmatrix} |a|^2 & 0 \\ 0 & |b|^2 \end{pmatrix} \Rightarrow \rho_A^2 \neq \rho_A$

$S_{VN,A} = -Tr\,\rho_A \ln \rho_A = -Tr\,\rho_A \begin{pmatrix} \ln|a|^2 & 0 \\ 0 & \ln|b|^2 \end{pmatrix} = -Tr \begin{pmatrix} |a|^2 \ln|a|^2 & 0 \\ 0 & |b|^2 \ln|b|^2 \end{pmatrix}$

$= -|a|^2 \ln|a|^2 - |b|^2 \ln|b|^2$

For $a = b = \frac{1}{\sqrt{2}}$ $S_{VN} = +\frac{1}{2}\ln 4 + \frac{1}{2}\ln 4 = 2\ln 2 \neq 0$ !

For $a = 1, b = 0$ $S_{VN} = 0$

What is the difference?

For $a = b = \frac{1}{\sqrt{2}}$ $|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle_A|\downarrow\rangle_B + \frac{1}{\sqrt{2}}|\downarrow\rangle_A|\uparrow\rangle_B$ A is entangled w/ B

$a = 1, b = 0$ $|\psi\rangle = |\uparrow\rangle_A|\downarrow\rangle_B$ A is not entangled w/ B

Note that after tracing over B, what we know is $\{p_i, |i\rangle_A\}$ very similar to the statistical case.

A big difference is $S_E(A) + S_E(B) \geq S_E(A \cup B)$, e.g.

$S_E = 0$

| A | B |
|---|---|
| $S_E \neq 0$ | $S_E \neq 0$ |